Basic Algorithms for Digital Image Analysis

Dmitrij Csetverikov   Collaborators

Eötvös Loránd University, Budapest, Hungary
csetverikov@sztaki.hu
http://visual.ipan.sztaki.hu/mitya/

Faculty of Informatics

Lecture 10 – Binary image processing II

Mathematical morphology

- **Morphology**
  - study of structure and form of animals, plants, or words, phrases
  - express structures and forms in terms of *structuring elements*
- **Mathematical morphology**
  - study of structure and form of images or other spatial structures
  - compare them to sliding *structuring element*
  - ⇒ hit objects with structuring element
  - ⇒ transform them to more revealing shapes

Two basic morphological operations

- Most morphological operations are defined in terms of two basic operations
  - **erosion**
  - **dilation**
- Basic definitions
  - B: **structuring element** with origin c
  - ⇒ discrete set of points of specific configuration aimed at particular operations
  - X: input digital image
  - B_x: translation of B so as to have origin c in point x of X
  - ⇒ result of morphological operation is assigned to point x of output image
- Figure (object) and ground
  - figure: nonzero (black in illustrations)
  - ground: zero (white in illustrations)

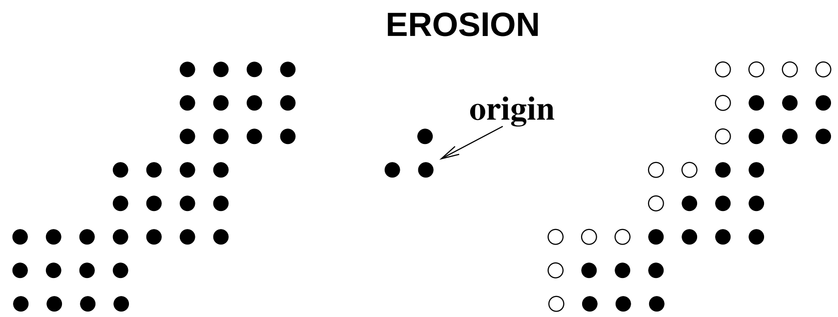
Erosion of X by B

- Set of all points x such that B_x is included in X

$$X \ominus B = \{x : B_x \subset X\}$$

- Nonzero output when
  - **all points** of structuring element coincide with nonzero points of image
- Zero output otherwise

Example of erosion



- Structural element is shown in centre
- Empty circle is **deleted point**
  - $\Rightarrow$ point x where B_x is not included in X

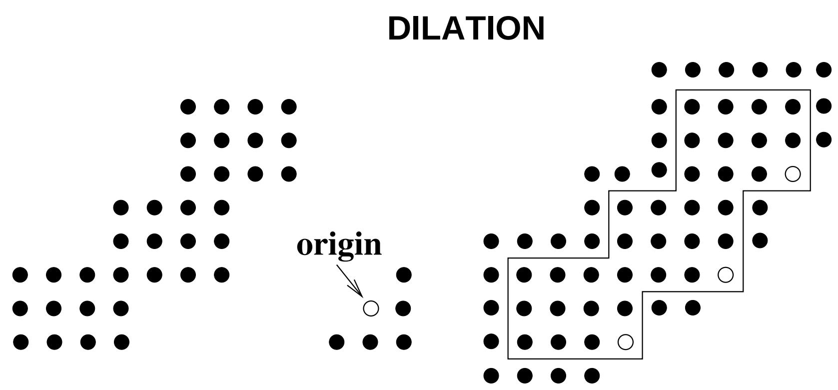
Dilation of X by B

- Set of all points x such that intersection of B_x and X is nonempty

$$X \oplus B = \{x : B_x \cap X \neq \emptyset\}$$

- Nonzero output when
  - **at least one point** of structuring element coincides with nonzero points of image
  - $\Rightarrow$ B_x hits X
- Zero output otherwise

Example of dilation



- Contour shows original object
- Empty circles are deleted points
- Despite deletion, resulting object is larger
  - $\Rightarrow$ but shifted due to shift of origin

Properties of erosion and dilation 1/5

- **Translation invariance**
  - shift of object leads to same shift in result
- Dilation is **commutative**

$$X \oplus B = B \oplus X$$

  - $B \oplus X$: B is treated as image, X as structuring element
  - $\Rightarrow$ similar to commutativity in linear filtering
- Erosion is **not commutative with image**

$$X \ominus B \neq B \ominus X$$

  - $\Rightarrow$ large object, small structuring element
  - $\Rightarrow$ empty $B \ominus X$, nonempty $X \ominus B$

Properties of erosion and dilation 2/5

- But order of erosions is arbitrary

$$(X \ominus A) \ominus B = (X \ominus B) \ominus A$$

- Erosion and dilation are **not inverses of each other**
  - $\Rightarrow$ erosion to empty set cannot be reversed

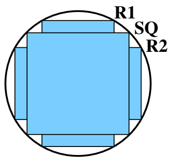
Properties of erosion and dilation 3/5

- **Distributivity in structuring element**

$$X \oplus (B \cup B') = (X \oplus B) \cup (X \oplus B')$$
$$X \ominus (B \cup B') = (X \ominus B) \cap (X \ominus B')$$

- Usage
  - structuring element can be represented/approximated by **union** of two or more simple elements
  - $\Rightarrow$ disc approximated by union of square and rectangles to speed up erosion
  - $\Rightarrow$ erosion by rectangle implemented like fast running filter

Disc approximated by square SQ and rectangles R1,R2



Properties of erosion and dilation 4/5

- **Distributivity in image**

$$(X \cap Y) \ominus B = (X \ominus B) \cap (Y \ominus B)$$
$$(X \cup Y) \oplus B = (X \oplus B) \cup (Y \oplus B)$$

- **Iteration**

$$(X \ominus B) \ominus B' = X \ominus (B \oplus B')$$
$$(X \oplus B) \oplus B' = X \oplus (B \oplus B')$$

  - dilation is **associative**, erosion is not
- Use iteration to **speed up erosion and dilation**
  - when structuring element can be represented as dilation of two or more smaller elements
  - $\Rightarrow$ erosion by 5×5 square is two erosions by 3×3 square

5 $\times$ 5 square is dilation of two 3 $\times$ 3 squares

$$\begin{matrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{matrix} = \begin{matrix}1&1&1\\1&1&1\\1&1&1\end{matrix} \oplus \begin{matrix}1&1&1\\1&1&1\\1&1&1\end{matrix}$$

Properties of erosion and dilation 5/5

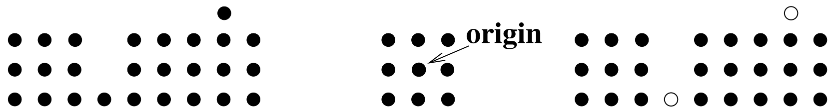
- **Increasing**
  - if $X \subset X'$, then
    $$X \ominus B \subset X' \ominus B$$
    $$X \oplus B \subset X' \oplus B$$
  - if $B \subset B'$, then
    $$X \ominus B' \subset X \ominus B$$
    $$X \oplus B \subset X \oplus B'$$
- **Duality** with respect to complement operation
  $$\underline{X} \oplus B^{\sim} = \underline{X \ominus B}$$
  - $\underline{X}$ is complement of X
  - $B^{\sim}$ is reflection of B with respect to its origin

Opening and closing

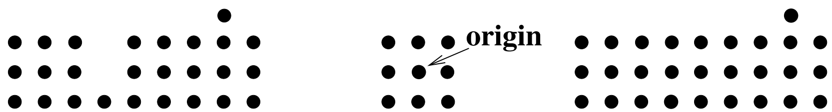
- **Opening** of set X by structuring element B
  $$X_B = (X \ominus B) \oplus B$$
  - smooths contours, removes small spots and sharp caps
  - can be used for object size distribution study
  - $\Rightarrow$ use openings with growing B
- **Closing** of set X by structuring element B
  $$X^B = (X \oplus B) \ominus B$$
  - fills up narrow channels and thin lakes
  - can be used for inter-object distance study
  - $\Rightarrow$ use closings with growing B

Examples of opening and closing

OPENING



CLOSING



Properties of opening and closing

- Opening and closing are **idempotent** operations

$$(X_B)_B = X_B$$
$$\left(X^B\right)^B = X^B$$

⇒ Do not apply twice with *same* B

- Opening and closing are **duals of each other**

$$\underline{(X_B)} = (\underline{X})^B$$
$$\underline{\left(X^B\right)} = (\underline{X})_B$$

⇒ Opening is (complement of) closing of complement

Hit-Miss: Matching binary patterns

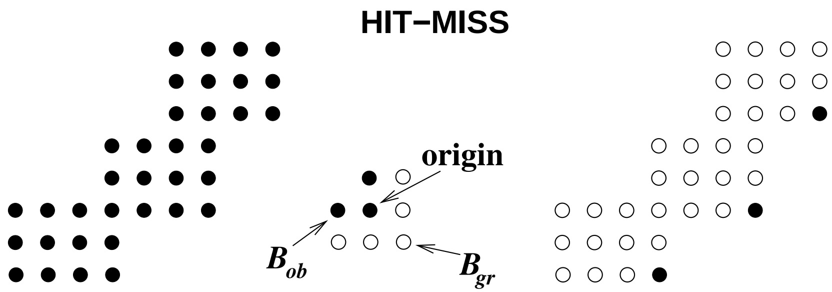
- Search for match or specific configuration
  - of **object and ground** pixels
- Specify both object and ground points of B
  - $B_{ob} \subset B$: object part of structuring element
  - $B_{gr} \subset B$: ground part of structuring element
- Hit-miss operator outputs object pixel when
  - B_{ob} matches object pixels **and**
  - B_{gr} matches ground pixels

Definition of hit-miss

$$X \otimes B \doteq \{x : B_{ob} \in X \text{ and } B_{gr} \in \underline{X}\} = (X \ominus B_{ob}) \cap (\underline{X} \ominus B_{gr})$$

- B_{gr} is treated as consisting of **object** pixels
  - they have to match **complement** $\underline{X}$
- Hit-miss in morphologic processing corresponds to pattern matching in greyscale images
- Other definitions of hit-miss also exist
  - **equivalent** to our definition
- Symbol $\odot$ is also used to denote the hit-miss

Finding corners with hit-miss



- Both figure and ground pixels of B must match
- Only finds corners of specific orientation and shape

  ⇒ rotate to find other corners of same shape

Boundary extraction

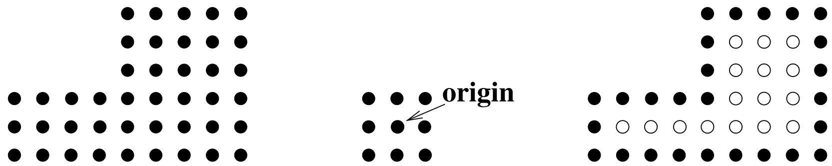
- **Boundary** of set X

$$\partial X = X \setminus (X \ominus G) \quad \text{with structuring element } G = \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

- Action
  - *interior points* are obtained by erosion
  - they are subtracted from original image
  - $\Rightarrow$ difference is boundary
- 3×3 square G approximates small *digital disc*
  - often used in morphological processing
  - larger 'discs' (squares) obtained using *iteration* property

Example of boundary extraction

BOUNDARY



- Interior points are obtained by erosion
- Then subtracted from original image
- Difference is boundary

Medial axis

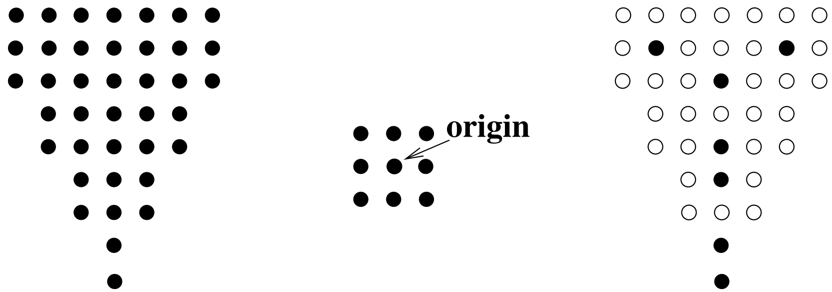
- **Medial axis** $S(X)$ of set X

$$S(X) = \bigcup_{n=0}^{n_{max}} (X \ominus nG) \setminus (X \ominus nG)_G \doteq \bigcup_{n=0}^{n_{max}} s_n(X)$$

- MA obtained as union of its parts $s_n(X)$
- Notation
  - structuring element G: 3×3 square ('disc')
  - n_{max}: maximum size after which X erodes down to zero
  - $(X \ominus nG)$: n^{th} iteration $(X \ominus G) \ominus G \ominus \cdots \ominus G$
  - $(X \ominus nG)_G$: opening of $(X \ominus nG)$ by G

Example of morphological medial axis

MEDIAL AXIS



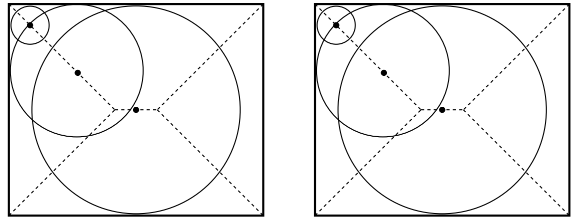
- Morphological MA is obtained as union of parts
- Parts may be separate from each other
  - $\Rightarrow$ contrary to continuous medial axis, morphological MA is not always connected

Restoring set X from its medial axis

$$X = \bigcup_{n=0}^{n_{max}} s_n(X) \oplus nG$$

- $Y \oplus nG$ is n^{th} iteration of $\oplus$ using G
- MA is detected as set of centres of maximal discs that
  - are contained in X **and**
  - touch boundary of X in two or more points
  - $\Rightarrow$ iterative *erosions* used
- To restore object from its MA take union of discs
  - centred on MA points **and**
  - having radii equal to contour distances
  - $\Rightarrow$ iterative *dilations* used

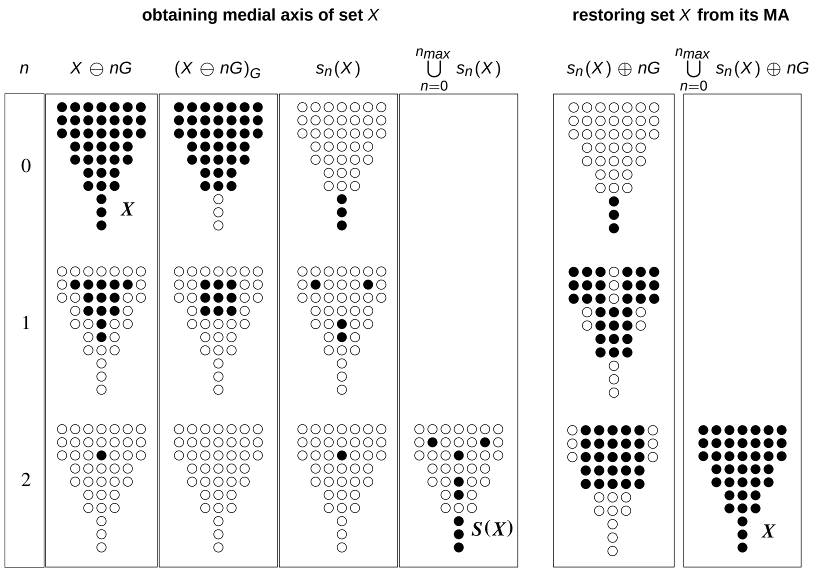
Meaning of morphological MAT and its inversion



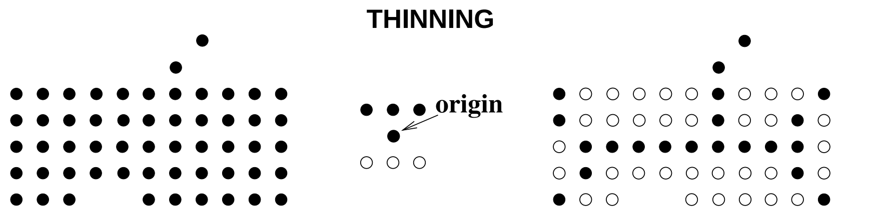
obtain MA　　restore shape from MA

- Obtain medial axis as locus of inscribed circles
  - **erosions by discs**
- Restore shape from MA
  - **dilations by discs**

Example of morphological medial axis



Thinning



- Uses hit-miss to reduce object to its skeleton
  - $\Rightarrow$ set of branches
- Iterative operation
  - at each iteration, *eight rotated versions* of structural element are used
  - results may depend on order in which rotated versions are applied

Comparison different thinnings

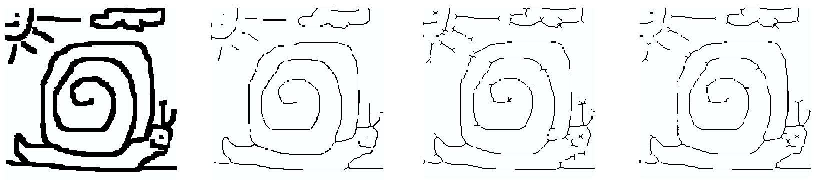
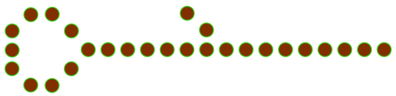


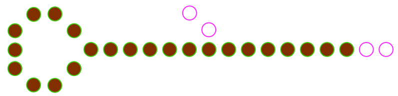
image conventional morpholog. 1 morpholog. 2

- Result of conventional thinning is better
- Morphological result
  - depends on order of rotations (1,2)
  - ⇒ sensitive to object orientation
  - contains small parasite branches

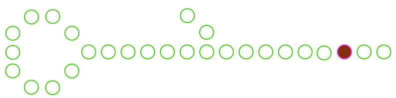
Example of morphological pruning



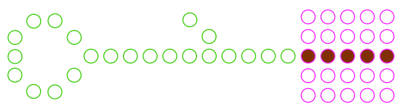
original object



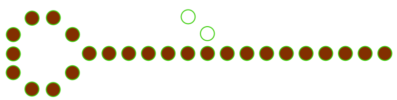
branches deleted



endpoint detected



endpoint grown back



pruned object

Morphological pruning of small branches

- Pruning eliminates small parasite branches of object
  - often applied after morphological MAT or thinning
- Morphological pruning involves several operations
  - recursively remove branches with rotating structural element
  - ⇒ number of recursions defines maximum length of branches to be removed
  - detect endpoints of remaining branches
  - recursively ‘grow back’ remaining branches by *geodesic dilation* using another element
  - ⇒ geodesic dilation is dilation limited to original set

Summary 1/2

- Morphological operations can be **extended to greyscale images**
- Morphology is used for
  - image pre-processing
  - ⇒ noise filtering, shape simplification
  - enhancing object structure
  - ⇒ skeletonisation, thickening
  - segmenting objects from ground
  - quantitative description of objects
  - ⇒ area, perimeter, projections, holes

Summary 2/2

- Morphological operations are best suitable
  - for processing images containing **many small objects**
  - for obtaining **statistical descriptions** of such images
  - ⇒ distributions of object sizes, perimeters, etc.
  - ⇒ fast hardware implementation available
- Not suitable for precise description of **large**, complex shapes
- Drawbacks
  - sensitivity to image orientation
  - approximate character of many results
  - ⇒ approximate MA, perimeter, etc.